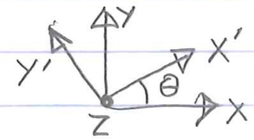
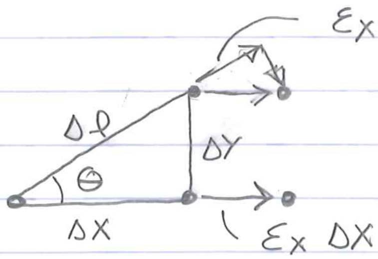
30. Transformation of strain

Consider the strain components ε_x, ε_y, γ_{xy} referred to x, y, z. We want to transform them to $\varepsilon_{x'}$, $\varepsilon_{y'}$, $\gamma_{x'y'}$ referred to x', y', z, where



First, develop a relationship for $\varepsilon_{x'}$.

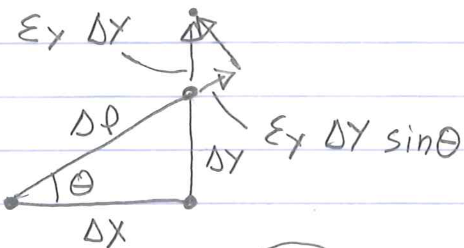
Impose ε_x



Contribution to $\varepsilon_{x'}$ is

$$\frac{\varepsilon_x \Delta X \cos\theta}{\Delta l} = \varepsilon_x \cos^2\theta$$

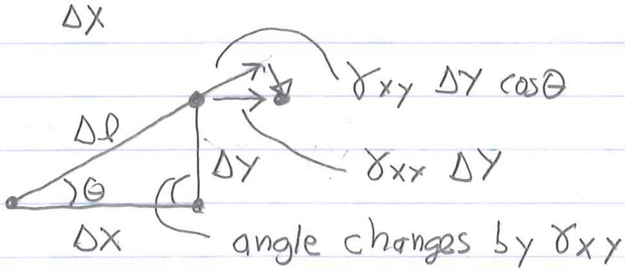
Impose ε_y



Contribution to $\varepsilon_{x'}$ is

$$\frac{\varepsilon_y \Delta Y \sin\theta}{\Delta l} = \varepsilon_y \sin^2\theta$$

Impose γ_{xy}



Contribution to $\varepsilon_{x'}$ is

$$\frac{\gamma_{xy} \Delta Y \cos\theta}{\Delta l} = \gamma_{xy} \cos\theta \sin\theta$$

The result is

$$\varepsilon_{x'} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy} \cos\theta \sin\theta$$

which holds if the rigid body rotations and strains ε_x, ε_y and γ_{xy} are small. This equation can be rewritten as

$$\varepsilon_{x'} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta + \frac{1}{2}\gamma_{xy} \sin 2\theta$$

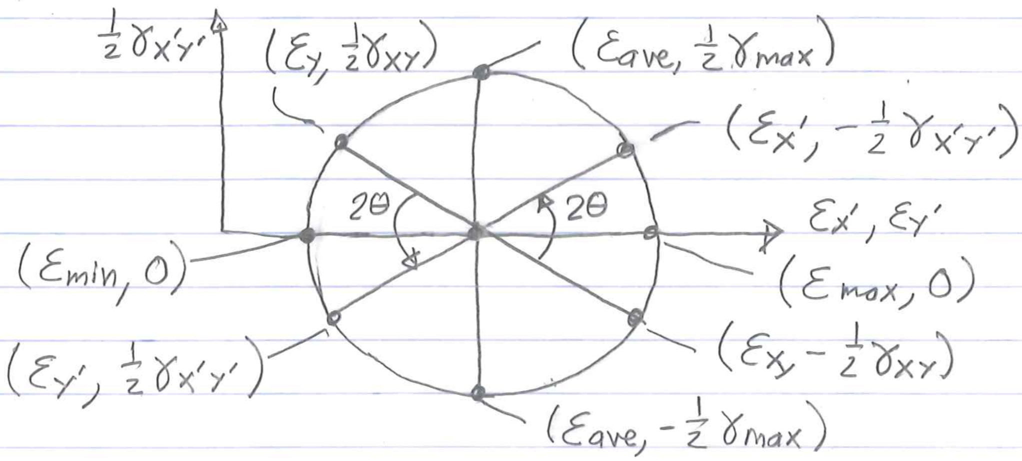
Similarly, the equations for $\varepsilon_{y'}$ and $\gamma_{x'y'}$ can be derived as

$$\varepsilon_{y'} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) - \frac{1}{2}(\varepsilon_x - \varepsilon_y)\cos 2\theta - \frac{1}{2}\gamma_{xy} \sin 2\theta$$

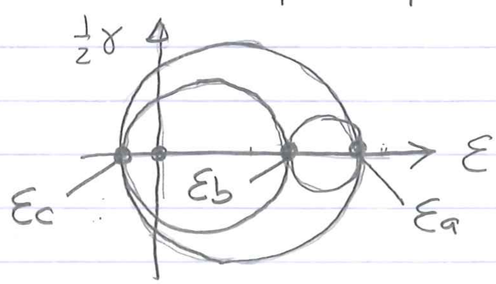
$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

These transformation equations for strain are the same as the ones for stress except that the σ's are replaced by ε's and the τ's are replaced by $\frac{1}{2}\gamma$'s. Note that the strain transformation equations are not affected by the presence of ε_z, γ_{xz} and γ_{yz}.

Strain transformation can be shown graphically with Mohr's circle of strain.

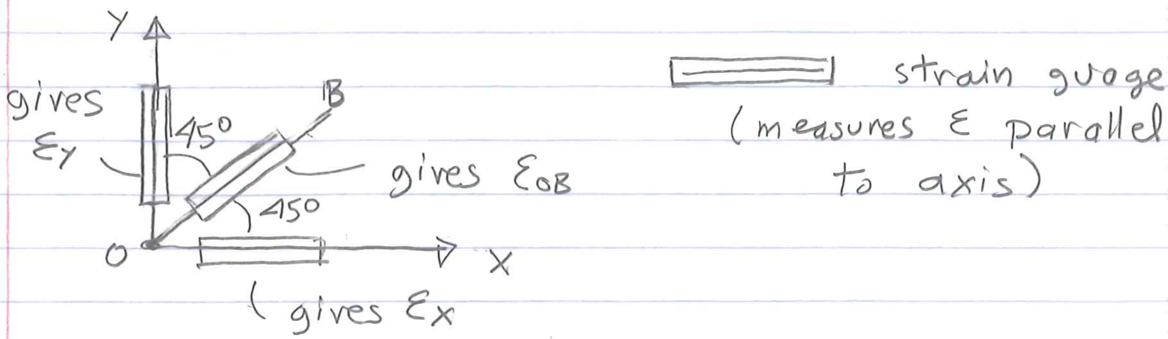


For a 3-d strain state, we have maximum principal strains ε_a, ε_b, ε_b where $\varepsilon_a \geq \varepsilon_b \geq \varepsilon_c$. ε_a is the maximum normal strain in any direction. ε_c is the minimum normal strain in any direction. $(\varepsilon_a - \varepsilon_c)$ is the maximum shear strain in any plane.



The principal axes for stress and strain are not necessarily coincident. They can be different for an anisotropic material.

Measuring strain with a strain rosette.



ε_x and ε_y are measured directly; we also want γ_{xy}. From the $\varepsilon_{x'}$ equation with $\theta = 45°$

$$\varepsilon_{OB} = \frac{1}{2}(\varepsilon_x + \varepsilon_y) + \frac{1}{2}\gamma_{xy}$$

So, $$\gamma_{xy} = 2\varepsilon_{OB} - \varepsilon_x - \varepsilon_y$$

Strain is often expressed as μ, ie, 10^{-6} (microstrain).